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VARIATION OF LATITUDE.*

BY W. J. HUSSEY.

The ordinary work of an engineer does not usually require him to take account of forces producing minute effects nor of quantities which are extremely small. Such forces and quantities are generally negligible when their magnitudes are small as compared with the uncertainties in the magnitudes of the other forces and quantities with which they are to be compared. In engineering structures the qualities of the materials employed and the friction of stationary and moving parts are ordinarily sources of quantities which are variable between wide limits and more or less indeterminate in their amounts. When this is the case, it is quite unnecessary to use a rigorous theoretical treatment or a refined practical application, so far as these uncertainties are concerned. But the engineer's work is not all of this character. Some of it requires rigorous treatment in theory and practice. When such is the case it is necessary to know and take account of all the influences which measurably affect the accuracy of the results. Numerous examples illustrating this might be cited. It is proper to mention here some of those belonging in common to the engineer and to the astronomer. It is so in geodetic work and in the determinations of meridians and important boundaries. These require a very high grade of engineering work combined with an equally high grade of astronomical work. In these cases the astronomical part consists largely in determining azimuths, longitudes and latitudes as accurately as possible. Each of these presents singularities peculiar to itself, and in the case of latitude

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the chief singularities relate to its variation. In the following paragraphs this subject will be briefly considered.

Before considering the phenomenon of the variation of latitude, as it appears from observation, it will be well to give, even at the risk of being somewhat technical, a few of the results which follow from the dynamical principles involved in the case of rotating bodies and then to see what the applications of these results are in the case of the Earth. For the present it will be assumed that the rotating body is perfectly rigid.

For any point of a rigid body there are three principal axes and three principal moments of inertia corresponding to them. A body rotating about a principal axis will preserve that axis as a permanent axis of rotation so long as the body is subject to the action of no disturbing force. A disturbing force will, however, change the position of the axis of rotation of the body. The amount of change will depend upon the degree of stability of the principal axis and upon the magnitude of the disturbing force. In case the rotation is *stable*, a small force will, at most, only cause a small deviation of the axis of rotation from a fixed straight line known as the invariable line. If, however, a small force eventually produces a very great change in the rotation, it is *unstable*. So far as has been, and, probably, can be determined, the rotation of the Earth is stable, and the actual deviation of the axis of rotation from the invariable line is very slight indeed. Still it is large enough to have a measurable influence on the results of astronomical observation, and as it is one of the chief causes of the periodic variation of latitude, we will proceed further with the results obtained from dynamical considerations.

When the rotation of a rigid body does not take place about a principal axis, the axis of rotation does not remain fixed in the body, but continually changes its position, even though the rotation is not disturbed by the action of any extraneous force. The line through the body coincident with the axis of rotation at any instant is at that instant the instantaneous axis of rotation. The movement of the instantaneous axis of rotation can in some cases be determined. The case of interest to us is that in which two of the principal moments of inertia about the center of gravity are equal and each nearly the same as the third one, and in which the axis of rotation is nearly though not quite in coincidence with the unequal principal axis. And for our case we may further assume that the rotation is not disturbed by the action of any ex-

traneous force. Under these conditions the invariable line and the instantaneous axis of rotation both move with reference to the body. They describe in it co-axial right cones, the common axis of which is the axis of unequal movement of the body. But the invariable line is fixed in space and consequently the axis of unequal moment really describes a right cone about it, and so also does the instantaneous axis of rotation. The semi-angles of the cones described about the axis of unequal moment by the invariable line and the instantaneous axis are connected with each other and with the principal moments of inertia by a very simple relation, from which, in the case of the Earth, it can be shown that these semi-angles are very nearly equal. The relation is

$$B \tan \alpha = A \tan \beta,$$

where α and β are respectively the angles made by the invariable line and the instantaneous axis of rotation with the axis of unequal moment and B the unequal and A one of the equal moments of inertia. Then α and β are the semi-angles of the co-axial cones and their difference the semi-angle of the cone described by the instantaneous axis about the invariable line.

The Earth is not exactly a spheroid of revolution, but it is so nearly so that this assumption will not lead to appreciable error. It is not a thoroughly rigid body, but in order to apply the above results it will, for the moment, be regarded as such. These conditions give two equal principal moments of inertia. Moreover, the rotation as assumed above is very approximately satisfied by it. The attractions of the Sun, Moon and planets do not act, as might at first be supposed, as forces disturbing that part of the motion now under consideration. Hence they do not invalidate the results given. They produce precession and nutation and although the effect now being considered is known as the Eulerian nutation, it really comes about by making an abstraction of the disturbing forces and regarding the Earth as simply set in rotation and left to itself. It is that part of the motion, analytically considered, which is derived from the complementary functions arising in the integration of the differential equations obtained in taking account of the action of the Sun and Moon in disturbing the rotation of the Earth. To obtain these complementary functions certain terms must be placed equal to zero, and this happens to be equivalent to making an abstraction of all extraneous forces. According to theory, then, the instantaneous axis

of rotation and the invariable line describe co-axial right cones in the Earth, of which the common axis is the axis of figure. And the rotation axis describes a right cone in space about the invariable line as an axis. Theory further shows that the axis of rotation will revolve about that of figure from west to east and in a period of

$$\frac{\sin \beta}{\sin (\alpha - \beta)}$$

days. In order to determine this theoretical period, α and β must be known, or, at least, since they are small and nearly equal, the ratio of either to their difference must be known. It can be shown, still assuming the Earth to be perfectly rigid, that this ratio cannot exceed 306 times either and that this is its approximate value. Hence, the theoretical period is about 306 days or about ten months. This is often referred to as "EULER'S ten-month period."

Such, in short and in part, is the theory of the rotation of the Earth as accepted until recently. But theory is not always correct and that it has not been so in this case has been demonstrated by Dr. CHANDLER in his recent important researches on the variation of latitude. And Professor NEWCOMB has pointed out a defect in the theory: it is in not taking account of the want of rigidity of the Earth and in wholly neglecting the fluidity of the oceans which surround it. These are important elements. Their effect is to lengthen the theoretical period of the revolution of the axis of rotation about that of figure. Thus, by assuming the Earth to be homogeneous and to have a rigidity equal to that of steel, a period of 441 days is obtained instead of 306 as required by a perfectly rigid Earth.

Latitude, as found by observation, is the complement of the angle included between a vertical line through the place of observation and the instantaneous axis. The vertical line is determined by the direction of gravity which may be assumed to be constant so long as the form of the Earth and the distribution of the materials composing it remain the same. From this it is evident that there will be a variation of latitude if the axis of rotation is not a fixed line in the Earth and that this variation will be periodic if the axis of rotation revolves regularly about that of figure. And it is further evident that the amplitude of variation will depend upon the angular separation of the two axes and that the phases

of variation will progressively move around the Earth. Places differing in longitude will, at any given time, exhibit different phases.

It has long been known that if such periodic variations exist and are of measurable magnitudes, they will afford the means of determining the position of the axis of rotation with reference to that of figure. Analytically considered, this requires, simply, the determination of the values of two constants of integration. By means of data furnished by observation attempts were made some years ago to find the values of these constants, with the result that both seemed to be zero, which would indicate a coincidence of the two axes. But such is not the case. Dr. CHANDLER has recently shown that there is a revolution of the axis of rotation from the west to the east in a period of 427 days and that the semi-amplitude of the variation of latitude due to it is about 0."12. He has also shown that there is another periodic variation, which will be spoken of later on.

That this periodic variation was not previously detected by those who attempted to discover it, PETERS, NYRÉN and others, is probably due to their too strict adherence to theory and to the 306-day period which it indicates. In cases like this where the quantities in question are small and likely to be confused with instrumental and other errors of observation, it cannot be expected that the solution of equations of condition will give results of a confirmatory character unless the assumptions which are involved in their formation approximate closely to the truth. Naturally, then, the use of a period of 306, instead of the true one of 427 days, would lead to inconclusive results. Dr. CHANDLER'S success, aside from the vast amount of painstaking labor which it has required, is due to his having perceived the inadequacy of existing theory to account for the observed phenomenon and to his skill in treating a difficult problem by strictly inductive methods. He tells us that he "deliberately put aside the teachings of theory, because it seemed high time that the facts should be examined by a purely inductive process; that the nugatory results of all attempts to detect the existence of the Eulerian period probably arose from a defect of the theory itself; and that the entangled condition of the whole subject should be examined afresh by processes unfettered by any preconceived notions whatever." And the problem which he proposed to himself was "To see whether it would not be possible to lay the numerous ghosts—in the shape

of various discordant residual phenomena pertaining to the determination of aberration, parallaxes, latitudes and the like—which had heretofore flitted elusively about the astronomy of precision, during the century; or to reduce them to a tangible form, by some simple, consistent hypothesis. It was thought that if this could be done, a study of the nature of the forces, as thus indicated, by which the Earth's rotation is influenced, might lead to a physical explanation of them."

In the course of his investigations, he has searched astronomical records for data bearing on the question and in doing so he has relied not merely on recent observations but has gone as far back as the time when BRADLEY was using his sextant at Wanstead in 1726. For somewhat more than a hundred years from the beginning of this period, the observations do not form a continous series, so that it has not been possible, so far, to trace satisfactorily the course of the variations of latitude which they plainly exhibit. But for the last fifty years the series is fairly continous and this has enabled results of great importance to be obtained. These will be given, but before doing so, it may be well to state that they have been obtained through a discussion of some thirty-three thousand observations, one-third of which were made in the southern hemisphere. Seventeen different observatories participated in making these observations, and four of these observatories belong to the southern hemisphere. In making them twenty-one different instruments and nine different methods of observation have been used.

The first result obtained has already been mentioned, the revolution of the axis of rotation about that of figure in 427 days. This produces a periodic variation of latitude having, as was finally determined, a semi-amplitude of 0."12. By a consideration of the results obtained at widely separated observatories, it was shown that this variation is neither instrumental nor local, but terrestrial, and it led at once to Professor NEWCOMB'S modification of the theory of the Earth's rotation. For a time, during the progress of the investigation, it seemed probable that this variation was subject to a considerable variation both in period and amplitude. But this is not the case. By a rearrangement of the data, the true character of the variations were disclosed and it was seen that this apparent variation is due to the superposition of another periodic variation. The latter has an annual period with its minimum and maximum values just before the vernal and

autumnal equinoxes respectively and its zeros just before the solstices. In amount its variation is not constant, but ranges from 0."04 to 0."20. The cause of this variation has not yet been ascertained, so that it is not possible to predict with certainty the effects which it may produce in the future. Its variations during the past fifty years are known in a general way. The minimum value of its range was reached some time between 1860 and 1880 and its larger value has prevailed before and since these dates.

The resultant variation of latitude due to these two causes is, of course, their sum. When they are at opposite phases, it may nearly disappear and when they are at the same phase and the annual term at its maximum, it may amount to nearly two-thirds of a second.

A rigorous formula representing the variations can not be given until the cause of the annual variation and its laws have been ascertained. Dr. CHANDLER has given a provisional formula, for the longitude of Greenwich, which closely represents the variations of the past fifty years, and it is probable that it will approximately represent them for a few years to come. In his formula the first term takes account of the variation due to the 427-day period and the second term that due to the annual period. His formula, after making a slight change in it so as to make it applicable at a place whose longitude is λ, is

$$\phi - \phi_0 = -0''12 \cos [\lambda + 0.^\circ 835 (t - T)] - r_2 \cos (\odot + 10^\circ)$$

where T and r_2, are computed by means of

$$T = 2406193 + 431\ E,$$
$$r_2 = 0.''047 + 0.''003\ \tau + 0.''00025\ \tau^2$$

and where the meanings of the symbols are as follows: ϕ and ϕ_0 are the instantaneous and mean latitudes of the place; T, the date at which the north pole of the Earth's figure passes the meridian of Greenwich; t, the date of observation; τ, the interval in years, positive after 1875; E, the number of completed revolutions between the date t and the adopted epoch 1875, November 1; and $\odot$ the longitude of the Sun. The interval $(t - T)$ is to be expressed in days. The first member $\phi - \phi_0$ symbolizes the variation of latitude at the time t. 1875, November 1 is the 2406193d day since the commencement of the Julian Period. This accounts for that number, it being the adopted epoch of the formula. A

somewhat simpler formula gives a considerably closer representation of the observations from 1862 to 1882. It is, for the meridian of Greenwich,

$$\phi - \phi_0 = -0.''125 \cos [0.^\circ 843 \, (t - 2406191)] - 0.''050 \cos (\odot + 10^\circ)$$

in which t is the number of days since the beginning of the Julian Period.

From the smallness of these variations it will be seen that the corrections to the latitude which they give will seldom need to be considered. It is only in the most accurate work that their being neglected will have any appreciable influence. As has been stated above, the problem of the variation of latitude is not yet fully solved. The cause of the annual term is yet unexplained. The question of the secular variation of latitude has been raised, but its existence has not yet been established, and even if it is, it is not likely to have any very sensible influence upon the results, either of the engineer or of the astronomer. From an astronomical point of view, the variation of latitude is important not in itself alone but also on account of the influence it has in vitiating the results of the astronomy of precision. It affects almost all kinds of absolute measurements. Equator point, equinoctical point, obliquity of the ecliptic, constant of aberration, stellar parallaxes, and absolute right ascensions and declinations of stars, may be mentioned as examples. And when the systematic errors introduced by it are eliminated, a conclusion is reached which is most gratifying to astronomers. It is that the degree of precision already attained in astronomical observation is far greater than had hitherto been supposed.

STANFORD UNIVERSITY, CALIFORNIA, April 8, 1893.

THE BRIGHT STREAKS ON THE MOON.

By J. M. SCHAEBERLE.

The remarkable appearance presented by the Moon's surface, more particularly with reference to the great light-reflecting power of certain peculiarly arranged areas, has given rise to various hypotheses concerning the cause of these features.

As a result of certain investigations at present under way,